

Correction Factors for Reactions involving $q\bar{q}$ Annihilation or Production

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Abstract

In reactions with $q\bar{q}$ production or $q\bar{q}$ annihilation, initial- and final-state interactions give rise to large corrections to the lowest-order cross sections. We evaluate the correction factor first for low relative kinetic energies by studying the distortion of the relative wave function. We then follow the procedure of Schwinger to interpolate this result with the well-known perturbative QCD vertex correction factors at high energies, to obtain an explicit semi-empirical correction factor applicable to the whole range of energies. The correction factor predicts an enhancement for $q\bar{q}$ in color-singlet states and a suppression for color-octet states, the effect increasing as the relative velocity decreases. Consequences on dilepton production in the quark-gluon plasma, the Drell-Yan process, and heavy quark production processes are discussed.

I. INTRODUCTION

The possibility of nuclear matter going through a phase transition into a deconfined quark gluon plasma (QGP) state during high-energy heavy-ion collisions makes these collisions the focus of intense experimental and theoretical research [1]. In these collisions, the constituents of the possible QGP or the partons of the colliding nucleons can react at varying energies. Their reaction products such as dileptons and photons provide the signals and the backgrounds for the detection of the quark-gluon plasma. The magnitudes of the signals depend on the cross sections for dilepton and photon production from the constituents of the plasma. The rates of approach to chemical equilibrium and thermal equilibrium of the plasma depend also on the cross sections for reactions among the constituents.

When quarks, antiquarks and gluons interact, they interact as constituents in the quark-gluon plasma or as partons in the colliding nucleon. However, the lowest-order Feynman diagrams and the next-to-leading order Feynman diagrams are the same for the basic reaction processes involving q , \bar{q} and gluons, whether the reaction takes place in the environment of the quark-gluon plasma or in the environment of partons in nucleon-nucleon collisions. In these basic reaction of quarks, antiquarks and gluons, the next-to leading order diagrams, including the initial- and final-state interactions and gluon radiations, give rise to large corrections to the lowest-order cross sections. It will be useful to develop an analytical semi-empirical correction factor for the basic reaction processes over the whole range of relative energies so that in the next level of approximation the basic lowest-order reaction cross sections of quarks, antiquarks and gluons can be corrected on the same footing. Additional effects and refinements such as the plasma screening and temperature can be added on in the future as our theoretical understanding is developed further.

For processes involving $q\bar{q}$ production and annihilation at high energies, it has been already generally recognized in perturbative QCD that the lowest-order Feynman diagrams give only an approximate description [8]- [23]. In the case of the Drell-Yan process for instance, the experimental cross section attributed to the process is about a factor of 2

to 3 greater than what one predicts by using the lowest-order Feynman diagram. A phenomenological K -factor is introduced by which the lowest-order results must be multiplied in order to bring the lowest-order QCD predictions into agreement with experiment. Higher-order QCD corrections to the Drell-Yan cross section, up to $O(\alpha_s)$ [11] and $O(\alpha_s^2)$ [13], have been worked out. These investigations show that in the high energy limit of massless quarks, the K factor can be well accounted for by including QCD corrections [11,13]. The most important contribution to the dilepton cross section due to the α_s -order (the next-to-leading-order) diagrams is the vertex correction at the $q\bar{q}\gamma^*$ vertex involving the initial-state interaction between q and \bar{q} and the gluon radiation from q and \bar{q} . According to [8–10], the vertex correction leads to a correction factor equal to $(1 + 2\pi\alpha_s/3)$. For a strong interaction coupling constant $\alpha_s = 0.3$, this vertex correction gives a factor of about 1.7. Additional contributions from the α_s -order Compton diagrams bring the Drell-Yan K -factor within the observed range of 1.6 to 2.8. For charm and heavy-quark production, the cross section for the lowest-order QCD was given in [15,16]. Higher-order QCD corrections to the cross sections and single-particle inclusive differential cross sections have also been obtained [17–20]. If one includes the next-to-leading-order of QCD, the K -factor is found to range from 2 to 3, depending on which heavy flavor is produced. The threshold behavior for heavy-quark production due to the distortion of the Q - \bar{Q} relative wave function has been examined in Refs. [2–6]. The correction is quite large near the threshold.

We focus in this work on basic reaction processes involving q and \bar{q} in the initial and final states. Other reactions involving gluons are important part of the dynamics in the plasma or partons and will be the subject of our subsequent studies. We investigate here the corrections to the tree-level cross sections involving the production and annihilation of $q\bar{q}$ at all relative energies. For low energies, we seek an analytical vertex correction factor arising from wave function distortion by virtue of the q - \bar{q} color potential. The distortion correction depends on α_s/v where v is the magnitude of the asymptotic relative velocity. It is the most important correction to the tree-level descriptions at energies where masses cannot be ignored, as has been earlier investigated for heavy quark production processes [2–6]. At

high energies, we use the well-known PQCD results which include the initial- or final-state interactions in addition to real gluon radiations. We shall seek an interpolation to join the vertex correction factor from low energies to these well-known perturbative QCD vertex correction factors at high energies. For the purpose of interpolation, we shall include the proper relativistic kinematics and follow the interpolation procedure suggested by Schwinger [30]. We obtain a simple analytical semi-empirical correction factor which can be used over the whole range of energies for both the annihilation and the production of a quark-antiquark pair.

This paper is organized as follows. In Section II, we first give a general discussion on the correction factor at low relative velocities arising from the distortion of the wave function at the point of $q\bar{q}$ annihilation or production. This correction factor is found to be an enhancement factor for the color-singlet states and a suppression factor for the color-octet states. The interpolation of this result with the known perturbative QCD correction factor at high energies provides correction factors which can be applied over the whole range of relative velocities. In Section 3, we show how we can use vertex correction factors to improve the lowest-order Drell-Yan cross sections. We discuss the use of these correction factors for dilepton production in the quark-gluon plasma in Section 4. The correction factors lead to an enhancement of the dilepton production probability in the plasma. In Section 5, we examine the $q\bar{q} \rightarrow Q\bar{Q}$ process, where both $q\bar{q}$ annihilation and $Q\bar{Q}$ production are found to be associated with suppressive correction factors. Section 6 summarizes the present discussions.

II. GENERAL CONSIDERATION OF THE CORRECTIVE FACTOR FOR $q\bar{q}$ ANNIHILATION AND PRODUCTION

What is the effective potential between the quark and the antiquark for continuum states that populate the production and annihilation vertices? The processes of production and annihilation are characterized by a region of very small relative q - \bar{q} separations, at a linear

distance of $\sim \alpha/\sqrt{s}$ for a virtual photon intermediate state and a distance of $\sim \alpha_s/\sqrt{s}$ for a virtual gluon intermediate state. The effective part of the $q\bar{q}$ interaction in this region of small relative separation associated with the annihilation or production process is the inverse- r , Coulomb-like term. The linear part of the potential, that serves to effect the binding and impose confinement, controls the large r behavior and needs not be considered in the collapsed spatial zone comprising the annihilation or production vertex. Therefore, we consider the interaction between a quark and an antiquark with an invariant mass $Q = \sqrt{s}$ as described by an effective Coulomb-type potential $A = (A_0, \mathbf{0})$ involving their relative coordinate r [2–6]

$$A_0(r) = -\frac{\alpha_{\text{eff}}}{r}, \quad (2.1)$$

where α_{eff} is the effective strong-interaction coupling constant, related to the strong interaction coupling constant α_s by the color factor C_f ,

$$\alpha_{\text{eff}} = C_f \alpha_s. \quad (2.2)$$

The running coupling constant is [8,9]

$$\alpha_s = \frac{12\pi}{(33 - 2n_f) \ln(Q^2/\Lambda^2)}, \quad (2.3)$$

where the flavor number n_f can be parametrized to be

$$n_f = \sum_{q=u,d,s,c,b,t} \left(1 - \frac{4m_q^2}{Q^2}\right)^{1/2} \theta\left(1 - \frac{4m_q^2}{Q^2}\right),$$

with the step function θ and the threshold behavior inferred from that of $q\bar{q} \rightarrow Q\bar{Q}$ cross section.

When q and \bar{q} are in their relative color-singlet states, the interaction is attractive and the color factor C_f is $4/3$ [8,1]. When q and \bar{q} are in their relative color-octet states, the interaction is repulsive and the color factor C_f is $-1/6$ [8].

It is worth mentioning that the notion of a potential is a useful concept, as one can infer from the success of the quasi-potential of Todorov [25] and Crater and van Alstine's

relativistic constraint dynamics to describe $q\bar{q}$ and e^+e^- as a relativistic two-body problem [26,27]. In these descriptions of constraint dynamics, the compatibility of the two-body equations of motion requires that the two-body potential must be expressed in terms of the relative four-coordinate x_\perp , orthogonal to the total four-momentum P . It is therefore convenient to work in the center-of-mass coordinate system, where $P = (\sqrt{s}, \mathbf{0})$ and the four-coordinate x_\perp is the relative coordinate r used in Eq. (2.1). In this coordinate system, the two-body system is described by an energy for relative motion given by

$$p^0 = \epsilon_\omega = \frac{s - 2m_q^2}{2\sqrt{s}}, \quad (2.4)$$

and the relativistic generalization of the reduced mass of the system is $m_\omega = m_q^2/\sqrt{s}$. The corresponding value of the on-shell relative momentum squared at $r \rightarrow \infty$ is

$$\mathbf{p}^2 = b^2(s) = \epsilon_\omega^2 - m_\omega^2 = \frac{s^2 - 4sm_q}{4s}. \quad (2.5)$$

(See Eqs. (2.13a), (2.13b), and (2.13c) of Ref. [27]).

A proper treatment of the relativistic two-body problem will require the solution of the two-body Dirac equation as formulated in Ref. [26,27] which is rather complicated. To gain a simple insight into the effect of the interaction between the quark and the antiquark, we shall treat the problem approximately as a single fermion spinor in the external field of Eq. (2.1) for which the solution is already known [24], with the stipulation of the correct relativistic kinematics to describe the relative motion between the quark and the antiquark. As is well-known, upon using the usual approximation as given by Ref. [24], the wave function which satisfies the correct boundary condition of an incident plane wave is

$$\psi = N e^{i\mathbf{P}\cdot\mathbf{r}} \left(1 - \frac{i}{2\epsilon_\omega} \boldsymbol{\alpha} \cdot \nabla\right) u F(i\xi, 1, i(|\mathbf{p}|r - \mathbf{p} \cdot \mathbf{r})), \quad (2.6)$$

where u is the spinor for a quark, F is the confluent hypergeometrical function, N is the normalization constant given by

$$|N|^2 = \frac{2\pi\xi}{1 - e^{-2\pi\xi}}, \quad (2.7)$$

and the parameter ξ (which can be positive or negative) is related to the coupling constant α_{eff} and the magnitude of the asymptotic relative velocity v by

$$\xi = \frac{\alpha_{\text{eff}}}{v}. \quad (2.8)$$

The magnitude of the asymptotic relative velocity v is the ratio of the momentum b to the energy ϵ_w in the system of relative coordinates. Following Todorov [25] and Eqs. (21.13a)-(2.13c) of Crater *et al* [27], the relative velocity for the quark and the antiquark in their center-of-mass system obtained by using Eqs. (2.4) and (2.5) is

$$v = \frac{(s^2 - 4sm_q^2)^{1/2}}{s - 2m_q^2}, \quad (2.9)$$

which gives $v \sim 2\sqrt{1 - 4m_q^2/s}$ when $\sqrt{s} \sim 2m_q$ and $v \rightarrow 1$ when $s \rightarrow \infty$. The quantity v is the asymptotic relative velocity obtained with the proper relativistic kinematics and differs from the quantity $\beta = \sqrt{1 - 4m_q^2/s}$ defined by Fadin *et al.* [6]. For the Drell-Yan $q\bar{q}$ annihilation processes or for the heavy-quark pair production processes, a quark and an antiquark are first annihilated into (or produced from) a virtual photon or a virtual gluon. As annihilation (or production) takes place in the vicinity of the region around $r = 0$, the probability for the annihilation process is proportional to the absolute square of the wave function at $r = 0$. When one averages over the spins of the quark, the square of the wave function at the origin in the case with interaction A_0 is

$$|\psi(0)|_{A_0}^2 = N^2 \left[1 + \left(\frac{\alpha_{\text{eff}}}{Ev} \right)^2 p^2 \frac{(1 - \cos \theta)}{2} \right], \quad (2.10)$$

where θ is the angle between the incident momentum \mathbf{p} and \mathbf{r} . After averaging over θ , we have

$$|\psi(0)|_{A_0}^2 = \frac{2\pi\xi}{1 - e^{-2\pi\xi}} \left(1 + \frac{\alpha_{\text{eff}}^2}{2} \right). \quad (2.11)$$

The above result was obtained by treating the $q\bar{q}$ system approximately as a fermion spinor in the field (2.1) of a spinless particle and the correction term $\alpha_{\text{eff}}^2/2$ inside the bracket arises from the spin of one of the two particles. When the spin of the other particle is taken into

account, there is an additional $\alpha_{\text{eff}}^2/2$ contribution, and the square of the wave function at the origin is modified to

$$|\psi(0)|_{A_0}^2 = \frac{2\pi\xi}{1 - e^{-2\pi\xi}}(1 + \alpha_{\text{eff}}^2). \quad (2.12)$$

On the other hand, in the perturbative expansion in which there is no interaction between the quark and the antiquark in the lowest-order approximation, the square of the wave function $|\psi(0)|_o^2$ is unity. Therefore, by considering the interaction between the quark and the antiquark as arising from an effective Coulomb-like interaction, the annihilation or production cross section is modified by the factor, which is a generalization of the ‘Gamow-Sommerfeld’ factor [28,29], given by

$$K = \frac{|\psi(0)|_{A_0}^2}{|\psi(0)|_o^2} = \frac{2\pi\xi}{1 - e^{-2\pi\xi}}(1 + \alpha_{\text{eff}}^2), \quad (2.13)$$

The expansion of the above K factor arising from the distorted wave function gives

$$K = \left[1 + \frac{2\pi\xi}{2} + \frac{(2\pi\xi)^2}{12} - \frac{(2\pi\xi)^4}{720} + \frac{(2\pi\xi)^6}{30240} - \frac{(2\pi\xi)^8}{1209600} + \dots \right] (1 + \alpha_{\text{eff}}^2), \quad (2.14)$$

which contains higher order terms with alternating signs. It is clear that for the case when $|\pi C_f \alpha_s / v|$ is much greater than 1, the convergence of the perturbation expansion as a power of α_s will be very slow. Many higher-order terms are needed to approach the full result Eq. (2.13). For that case, a perturbative treatment of the correction factor is not useful and a non-perturbative treatment such as presented here in terms of wave-function distortion represented by Eq. (2.13) is a meaningful concept.

The correction factor Eq. (2.13) has been obtained by considering only the exchange of virtual gluons between the quark and the antiquark. The emission of real soft gluons is not taken into account. Because radiative corrections involving the emission of real gluons is unimportant for $q\bar{q}$ systems with small relative velocities, the results of Eq. (2.13), without the inclusion of real soft gluons, is applicable to the region of low relative velocities.

At very-high relative velocities, besides the distortion effect arising from the initial-state and final-state interactions between the quark and the antiquark, it is necessary to include

the radiation of real gluons. In that case, the amplitude from the emission of soft gluons interferes with the amplitude arising from the exchange of a virtual gluon between the quark and the antiquark. At very high relative velocities for which the masses of the quarks can be neglected, perturbative QCD calculations have been carried out by including the exchange of a virtual gluon between the quark and the antiquark and the emission of real gluons from the two particles [11,13,8,9]. These high-energy vertex correction K -factors are different for $q\bar{q}$ annihilation or for $q\bar{q}$ production. As the vertex correction gives the dominant contribution, we shall consider the approximation where the phenomenological correction factor K is given just by the vertex correction. Up to the first order in α_s in the high-energy limit of massless quarks, the vertex K -factor for $q\bar{q}$ annihilation is [11]

$$\begin{aligned} K &= 1 + \pi\alpha_{\text{eff}}\left(\frac{1}{2\pi^2} + \frac{5}{6}\right) \\ &= 1 + \pi\alpha_{\text{eff}} \times 0.884, \end{aligned} \tag{2.15}$$

which is about 2 for a typical $\alpha_{\text{eff}} = 0.4$ for color-singlet states. The vertex correction K -factor for $q\bar{q}$ production in the $e^+e^- \rightarrow q\bar{q}$ process is [8,9]

$$K = 1 + \frac{\alpha_s}{\pi}, \tag{2.16}$$

which is about 1.1 for $\alpha_{\text{eff}} = 0.4$. On account of the fact that the singlet color factor for e^+e^- annihilation is $4/3$, the correction factor for $q\bar{q}$ production is

$$K = 1 + \frac{3\alpha_{\text{eff}}}{4\pi}. \tag{2.17}$$

Thus, for $q\bar{q}$ in a color-singlet state at high energies, the high-order QCD corrections are large for $q\bar{q}$ annihilation and small for $q\bar{q}$ production. In contrast, at low relative velocities, Eq. (2.13) gives equally large QCD corrections for $q\bar{q}$ annihilation or production.

Knowing the two limits of the correction factors from Eqs. (2.13), (2.15), and (2.17), we can follow the interpolation procedure suggested by Schwinger [30] and used by Barnett *et al.* [3]. We can obtain a semi-empirical correction factor which can be used for the whole range of relative velocities. For $q\bar{q}$ annihilation in the initial state, we define a function $f^{(i)}(v)$ by

$$f^{(i)}(v) = \alpha_{\text{eff}} \left[\frac{1}{v} + v \left(-1 + \frac{1}{2\pi^2} + \frac{5}{6} \right) \right]. \quad (2.18)$$

For $q\bar{q}$ production in the final state, we define a function $f^{(f)}(v)$ by

$$f^{(f)}(v) = \alpha_{\text{eff}} \left[\frac{1}{v} + v \left(-1 + \frac{3}{4\pi^2} \right) \right]. \quad (2.19)$$

Associated with the annihilation or production of a $q\bar{q}$ pair at a center-of-mass energy \sqrt{s} , we propose a semi-empirical vertex K -factor as

$$K^{(i,f)}(q) = \frac{2\pi f^{(i,f)}(v)}{1 - \exp\{-2\pi f^{(i,f)}(v)\}} (1 + \alpha_{\text{eff}}^2), \quad (2.20)$$

where the flavor label q in $K^{(i)}(q)$ is included to indicate that $K^{(i)}$ depends on the quark mass m_q , the superscript (i) and (f) denotes $q\bar{q}$ initial-state annihilation and final-state production respectively, and the magnitude of the relative velocity v is given by Eq. (2.9). This vertex correction factor agrees with the results of Eq. (2.13) for low relative velocities. For high relative velocities and $|\pi\alpha_{eff}| < 1$, Eq. (2.20) gives

$$K^{(i,f)} \rightarrow 1 + \pi f^{(i,f)}, \quad (2.21)$$

and Eq. (2.20) agrees with the well-known results of Eqs. (2.15) and (2.17) from perturbative QCD for high relative velocities, up to the first order in α_s . Thus, Eq. (2.20) can provide a good description of the vertex corrections for $q\bar{q}$ systems over the whole range of relative velocities, for $q\bar{q}$ annihilation or $q\bar{q}$ production. Because the vertex correction gives the dominant QCD correction, the vertex correction factor (2.20) can be used to give the corrections to lowest-order QCD results when a $q\bar{q}$ pair is annihilated or produced. It could equally well be used with a subscript ‘v’ to identify its association with the vertex correction and distinguish it from the usual phenomenological K factor that usually quantifies the QCD corrections in the massless limit only. However, it has been retained as K for compactness and to allow easy applicability of the correction.

III. DRELL-YAN PROCESSES

As an example of the application of the correction factor $K^{(i,f)}$ of Eq. (2.20), we can use it to improve theoretical tree-level estimates of the Drell-Yan cross section. In the Drell-Yan process, a $q\bar{q}$ pair is annihilated in the initial state, and the appropriate correction factor is $K^{(i)}$. As the virtual photon in the $q\bar{q}\gamma^*$ vertex selects the color-singlet combination of the annihilating $q\bar{q}$ pair, the corresponding color factor C_f is $4/3$, and the effective coupling constant needed to calculate $K^{(i)}$ in Eq. (13) is $\alpha_{\text{eff}} = 4\alpha_s/3$.

If only the lowest theoretical estimate $d\sigma^{\text{LO}}/dQ^2 dy$ is available, an improved determination of the Drell-Yan cross section for $q\bar{q} \rightarrow l^+l^-$ can be provided by

$$\frac{d\sigma}{dQ^2 dy} = K^{(i)}(q) \frac{d\sigma^{\text{LO}}}{dQ^2 dy}.$$

In Fig. 1, we show $K^{(i)}(q)$ as a function of the invariant mass $Q = \sqrt{s}$ of the dilepton pair calculated with $\Lambda = 0.3$ GeV. For a $q\bar{q}$ pair in a color-singlet state at high relative velocities, the $K^{(i)}$ factor is about 2.2 which agrees with the experimental value of 1.6 to 2.8 [9] in the Drell-Yan process. For color-singlet states with low relative velocities, the correction factor is quite large. The correction factor for a $q\bar{q}$ pair in the color-octet state approaches 0.9 as \sqrt{s} increases and it deviates from this constant behavior at low relative velocities when the energy is near the $q\bar{q}$ mass threshold.

When the next-to-leading order results, including other diagrams such as Compton diagrams are available, the estimate of the Drell-Yan cross section can be improved to include the distortion corrections as well. By following a procedure which was suggested by Harris and Brown [31], we can avoid double counting.

Accordingly, the next-to-leading order results of Kubar *et al.* for the Drell-Yan cross section can be modified to include distortion effects by using the K -factor as given by Eq. (2.13) prior to interpolation with the high energy QCD corrections. Using most of the notation of Kubar *et al.*, the cross section for the production of an l^+l^- pair with an invariant mass squared Q^2 and a rapidity y for the collision of two hadrons is

$$\frac{d\sigma}{dQ^2 dy} = \frac{d\sigma^{\text{LO}}}{dQ^2 dy} + \frac{d\sigma^{\text{A}}}{dQ^2 dy} + \frac{d\sigma^{\text{C}}}{dQ^2 dy}, \quad (3.1)$$

with

$$\frac{d\sigma^i}{dQ^2 dy} = \int dt_1 dt_2 \sum_f \frac{d\hat{\sigma}^i}{dQ^2 dy} Q_f(t_1, t_2), \quad (3.2)$$

where $i = \text{LO}, \text{A}, \text{C}$ represent the contributions from the lowest order Drell-Yan diagram, the annihilation diagrams, and the Compton diagrams, respectively. The function Q_f is the product of the quark and antiquark structure functions of the colliding hadrons,

$$Q_f(t_1, t_2) = q_{10}^f(t_1) \bar{q}_{20}^f(t_2) + \bar{q}_{10}^f(t_1) q_{20}^f(t_2), \quad (3.3)$$

and f is the flavor label.

When the distortion due to the wave function is taken into account, the result of Kubar *et al* [11] can be modified to be

$$\begin{aligned} & \frac{d\hat{\sigma}^{\text{LO}}}{dQ^2 dy} + \frac{d\hat{\sigma}^{\text{A}}}{dQ^2 dy} \\ &= \frac{4\pi\alpha^2}{9Q^2 s} \left(\frac{e_f}{e} \right)^2 \left[\delta(t_1 - x_1) \delta(t_2 - x_2) K^{(i)} \right. \\ & \quad \times \left\{ 1 + \frac{|\alpha_{\text{eff}}|}{2\pi} \left[-\frac{3}{2} \ln \frac{x_1 x_2}{(1-x_1)(1-x_2)} + 2 \ln \frac{x_1}{1-x_1} \ln \frac{x_2}{1-x_2} \right] \right\} \\ & \quad + \frac{|\alpha_{\text{eff}}|}{2\pi} \delta(t_2 - x_2) \left(\frac{t_1^2 + x_1^2}{t_1^2 (t_1 - x_1)_+} + \ln \frac{2x_1(1-x_2)}{x_2(t_1 - x_1)} + \frac{3}{2(t_1 - x_1)_+} - \frac{2}{t_1} - \frac{3x_1}{t_1^2} \right) + (1 \leftrightarrow 2) \\ & \quad \left. + \frac{|\alpha_{\text{eff}}|}{\pi} \left(\frac{G^{\text{A}}(t_1, t_2)}{[(t_1 - x_1)(t_2 - x_2)]_+} + H^{\text{A}}(t_1, t_2) \right) \right], \quad (3.4) \end{aligned}$$

where $K^{(i)}$ is now given by Eq. (2.13), the distributions $1/(t-x)_+$, and $1/[(t_1 - x_1)(t_2 - x_2)]_+$ are defined by

$$\int_x^1 dt \frac{f(t)}{(t-x)_+} = \int_x^1 dt \frac{f(t) - f(x)}{t-x},$$

and

$$\int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \frac{f(t_1, t_2)}{[(t_1 - x_1)(t_2 - x_2)]_+} = \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \frac{f(t_1, t_2) - f(t_1, x_2) - f(x_1, t_2) + f(x_1, x_2)}{(t_1 - x_1)(t_2 - x_2)}.$$

In another perturbative treatment of the Drell-Yan process, an estimate of the first-order vertex correction leads to a vertex K factor of the form [8–10]

$$K' = 1 + 2\pi\alpha_s/3, \quad (3.5)$$

and the generalization to higher-orders of the form

$$K' = e^{2\pi\alpha_s/3}. \quad (3.6)$$

However, this result has been obtained only in the high energy limit and may not be applied to the cases of low dilepton invariant masses. Furthermore, the vertex correction factor of Eq. (3.5) appears to be only a part of the whole vertex correction factor of Eq. (2.15).

Combined with the running coupling constant, the use of the correction factor Eq. (2.20) allows us to probe lower dilepton invariant masses and the important low- x region with better justification than massless limits.

IV. THERMAL DILEPTON PRODUCTION

The importance of the possibility of hadron matter going through a phase transition into a deconfined quark-gluon plasma state, during high-energy heavy-ion collisions has been highlighted earlier [1]. While the basic process leading to the formation of dileptons is the same as that for the Drell-Yan process, the distributions and the characteristics of the annihilating quarks are quite different for the two cases. The final state dileptons are expected to carry information on the environment of the annihilating quark-antiquark pair and therefore serve as a probe of the thermodynamical state of the quark-gluon plasma.

The rate for the production of dileptons with an invariant mass Q per unit four-volume in a thermalized quark-gluon plasma is sensitive to the temperature T of the system and can be written in the form [32,1]

$$\frac{dN_{l^+l^-}}{dQ^2 d^4x} \sim N_c N_s^2 \sum_{q=u,d,s,\dots} \left(\frac{e_q}{e}\right)^2 \frac{\sigma_q(Q)}{2(2\pi)^4} Q^2 \sqrt{1 - \frac{4m_q^2}{Q^2}} T Q K_1\left(\frac{Q}{T}\right), \quad (4.1)$$

where $\sigma_q(Q)$ is the lowest order $q\bar{q} \rightarrow l^+l^-$ cross section at the center-of-mass energy Q given by [1]

$$\sigma_q(Q) = \frac{4\pi}{3} \frac{\alpha^2}{Q^2} \left(1 - \frac{4m_q^2}{Q^2}\right)^{-\frac{1}{2}} \sqrt{1 - \frac{4m_l^2}{Q^2}} \left(1 + 2\frac{m_q^2 + m_l^2}{Q^2} + 4\frac{m_q^2 m_l^2}{Q^4}\right), \quad (4.2)$$

m_l , m_q are the rest masses of the lepton l and the quark q respectively, and K_1 is the modified Bessel function of first order.

In order to be meaningful experimental signals for the detection of QGP states, the thermal dileptons must be clearly delineated from other sources of dileptons, particularly the Drell-Yan background. It becomes important therefore to have good estimates of the dilepton production from the different sources at matching levels of accuracy. It is hence necessary to modify equation (4.1), to incorporate the higher-order QCD corrections into the rate for dilepton production in the quark-gluon plasma.

The interpolation from low to high relative velocities extends the usefulness of the correction factors Eq. (2.20). Therefore, we may introduce the same correction factor $K^{(i)}$ as used in the DY case to account for high order QCD effects. The modified rate of dilepton production becomes

$$\frac{dN_{l^+l^-}}{dQ^2 d^4x} \sim N_c N_s^2 \sum_{q=u,d,s,\dots} \left(\frac{e_q}{e}\right)^2 \frac{K^{(i)}(q) \sigma_q(Q)}{2(2\pi)^4} Q^2 \left(1 - \frac{4m_q^2}{Q^2}\right)^{\frac{1}{2}} T Q K_1\left(\frac{Q}{T}\right). \quad (4.3)$$

The virtual photon annihilation mode selects the annihilating $q\bar{q}$ pair to be in the color-singlet states and thus the interaction is attractive. The distortion effect leads to an enhancement factor $K^{(i)}$ as shown in Fig. 1. Because the coupling constant increases and the relative velocity decreases as the invariant mass decreases, the correction factor rises considerably with the decrease of invariant mass. The effect of the higher order QCD corrections is to enhance the tree-level dilepton cross section by a factor of about 4 at $\sqrt{s} = 1$ GeV, and by a factor of about 3 at $\sqrt{s} = 2 - 3$ GeV. Higher order QCD corrections will increase the strength of the dilepton signal and may serve to enhance the prospects for its detection in ongoing and planned experiments [33].

It may be remarked that in the literature, a correction K factor has sometimes been used for thermal production as $[1 + (\alpha_s/\pi)(1 + aT^2/Q^2)]$ (Eq. (5.1) of Ref. [21]), which coincides approximately with $K^{(f)}$ that includes the first-order correction for the reverse $l^+l^- \rightarrow q\bar{q}$

process [8], but is smaller than the correction factor $K^{(i)}$. Recent calculations, for the first order correction, using the thermal mass, predict a much larger K factor similar to our $K^{(i)}$ in magnitude [22].

The question of screening in the quark gluon plasma requires a comment at this stage. In the plasma, the color charge of the constituents of the plasma is subject to Debye screening which is characterized by the Debye screening length λ_D depending on the temperature T . For the quark-gluon plasma with a flavor number N_f and $N_c = 3$, the lowest-order perturbative QCD theory gives [34]

$$\lambda_D(\text{PQCD}) = \frac{1}{\sqrt{(\frac{N_c}{3} + \frac{N_f}{6})g^2} T}. \quad (4.4)$$

For a coupling constant $\alpha_s = g^2/4\pi = 0.5$ and $N_f = 3$, the Debye screening length at a temperature of 200 MeV is about $\lambda_D \sim 0.4$ fm. On the other hand, the electromagnetic annihilation into dileptons occurs within an extremely collapsed space zone characterized by the linear $q\bar{q}$ distance $\sim \alpha/\sqrt{s}$ which should be compared with the the Debye screening radius λ_D in the plasma. As the annihilation distance α/\sqrt{s} is about 0.0029 fm for the annihilation of a light quark-antiquark pair at 0.5 GeV, and is much smaller than the Debye screening length λ_D , the interaction between the quark and the antiquark is not much affected by Debye screening in the region where annihilation occurs. Therefore, we expect that our correction factor for dilepton production by $q\bar{q}$ annihilation in the plasma will not be modified much by the addition of Debye screening corrections.

V. HEAVY-QUARK PRODUCTION

Heavy quark production in nuclear collisions, whether routed through intermediate QGP states or not, occurs either by $q\bar{q}$ annihilation ($q\bar{q} \rightarrow Q\bar{Q}$) or by gluon-gluon fusion, ($gg \rightarrow Q\bar{Q}$). In the first case, the final state heavy quark-antiquark pair are required to be in a color-octet outgoing state, due to the color-octet nature of the intermediate gluon states, while in the latter, they may emerge in either the color-singlet or color-octet states. We

shall limit our investigation in this paper to the former process, and discuss the gluon-gluon process in a future publication.

The cross section for $q\bar{q} \rightarrow Q\bar{Q}$ can be written at tree-level as [1]

$$\sigma(s) = \frac{8\pi\alpha_s^2}{81s} \left(1 - \frac{4m_q^2}{s}\right)^{-\frac{1}{2}} \sqrt{1 - \frac{4m_Q^2}{s}} \left(1 + 2\frac{m_q^2 + m_Q^2}{s} + 4\frac{m_q^2 m_Q^2}{s^2}\right). \quad (5.1)$$

In the $q\bar{q} \rightarrow g^* \rightarrow Q\bar{Q}$ process, there are two strong-interaction vertices, $q\bar{q}g^*$ and $g^*Q\bar{Q}$. Both of these require QCD corrections, along with the other radiative corrections involving the external quark lines and their connections with the virtual gluon propagator.

In the present work, we attempt to correct for the two interaction vertices simultaneously, to include the wave-function distortion effect at low relative velocities and additional real gluon radiation at high relative velocities, in the same way as in our earlier corrections for the DY process.

The effective interaction between the quark and the antiquark can be written for each vertex as in section II, and we can similarly write a K factor for each vertex. The first vertex, $q\bar{q}g^*$, occurs with the annihilation of a $q\bar{q}$ pair in the initial state and is associated with the $K^{(i)}(q)$ correction factor. The vertex is identical to the DY case except for the replacement of the virtual photon by the virtual gluon. Because of the color-octet nature of the intermediate gluon, the quark-antiquark pair must likewise be in the color-octet state. The correction factor $K^{(i)}(q)$ for the annihilation of this color-octet state corresponds to an effective repulsive potential with the color factor C_f equal to $-1/6$. As we note from Fig. 1, it has the value of about 0.9 at high energies.

The second vertex, $g^*Q\bar{Q}$, occurs with the production of the $Q\bar{Q}$ pair in the final state and is associated with the $K^{(f)}(Q)$ correction factor given by Eq. (2.20), having the appropriate function $f^{(f)}(v)$ of Eq. (2.20). Since the color factor is same as the first vertex, this correction is also suppressive.

Corrected for wave function distortion at both vertices, the cross section (5.1) is amended to

$$\sigma(s) = K^{(i)}(q)K^{(f)}(Q)\frac{8\pi\alpha_s^2}{81s} \left(1 - \frac{4m_q^2}{s}\right)^{-\frac{1}{2}} \sqrt{1 - \frac{4m_Q^2}{s}} \left(1 + 2\frac{m_q^2 + m_Q^2}{s} + 4\frac{m_q^2 m_Q^2}{s^2}\right). \quad (5.2)$$

In the range of invariant mass close to the masses of the heavy quarks in question, the ideal analysis envisages correct inclusion of the heavy quark masses. The vertex factor $K^{(f)}(Q)$ accordingly acquires a sensitivity to the mass. In Fig. 2, we display the correction factor $K^{(f)}$ for the production of $q\bar{q}$ pairs of various flavors in color-singlet or color-octet states, calculated with $\Lambda = 0.3$ GeV. The values of the correction factors are constants at relativistic relative velocities, but begin to depart from the constant values as the invariant mass approaches the threshold, corresponding to lower relative velocities between the quark and the antiquark. Near the threshold, the enhancement of the color-singlet states due to distortion is much more substantial than the corresponding suppression of the production of color-octet states.

How does the Debye screening affect the correction factor? The annihilation or production cross section in the $q\bar{q} \rightarrow Q\bar{Q}$ process involves the strong coupling constant and is associated with an annihilation or production length of the order of α_s/\sqrt{s} (*cf.* Eq. (5.1)) which should be compared with the Debye screening length λ_D of about 0.4 fm at $T = 200$ MeV. The effect of Debye screening is important when $\alpha_s/\sqrt{s} \gg \lambda_D$; it is unimportant when $\alpha_s/\sqrt{s} \ll \lambda_D$. Accordingly, for quark-antiquark production involving the annihilation or the production of light $q\bar{q}$ pairs through a virtual gluon intermediate state, the effect of Debye screening is important when $\sqrt{s} \ll 0.25$ GeV, assuming a coupling constant of $\alpha_s = 0.5$. It is unimportant when $\sqrt{s} \gg 0.25$ GeV. Thus, the correction factors can be used for $q\bar{q}$ pairs with energies \sqrt{s} much greater than 0.25 GeV.

VI. CONCLUSIONS AND DISCUSSIONS

At low energies of relative motion, the quark-antiquark relative wave function becomes distorted in the field of the color potential acting between a quark and its antiquark partner [2–6]. The distortion is strong in the vicinity of their zero separation and leads to a significant modification of the cross section for their annihilation or production. When the quark-antiquark pair is in its color-singlet state, the interaction is highly attractive, and

the cross section is much enhanced. On the other hand, when the quark-antiquark pair is in its color-octet state, the interaction is repulsive and the cross section is suppressed. This enhancement or suppression can be quantified in terms of a vertex correction factor K . The distortion effect does not depend on whether the quark-antiquark pair is annihilated or produced. At high energies of relative motion such that the quark masses can be neglected, the addition of real soft gluon emission leads to well-known PQCD vertex correction factors which are different for $q\bar{q}$ annihilation or production [8]. Following the interpolation procedure of Schwinger to connect the vertex correction factor for low relative velocities, (obtained from wave function distortion), to the vertex correction factor for high relative velocities, (obtained from perturbative QCD), we can interpolate the correction factor to make it applicable to the whole range of relative velocities. The correction factor enables us to correct tree-level QCD calculations, when a $q\bar{q}$ pair is annihilated or produced. Its inclusion improves both our understanding of the underlying physics and the accuracy of the reaction cross sections.

We wish to emphasize here the usefulness of our unified description to correct the lowest-order results in different environments. As our $K^{(i,f)}$ does not depend on the structure functions or constituent distributions in $q\bar{q}$ annihilation or the mode of $q\bar{q}$ production, it does not have to be recalculated for every individual environment. It is sufficient to ensure that the process considered is described by basic diagrams involving $q\bar{q}$ annihilation or production.

As a test of the reliability of the correction factor Eq. (2.20), we apply the correction factor for $q\bar{q}$ production to study the ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. For heavy quark production in the $e^+e^- \rightarrow \gamma^* \rightarrow Q\bar{Q}$ process, Q and \bar{Q} are produced in the color-singlet final state. The tree-level cross section must be multiplied by $K^{(f)}$ of Fig. 2 to take into account higher-order QCD corrections. The results of Fig. 2 shows a very large enhancement near the threshold of heavy-quark production. This large enhancement is indeed observed and the results Fig. 2 give good agreement with experimental ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ [35]. The good agreement lends support to the usefulness of Eq. (2.20) for its application to other reaction processes.

Accordingly, one can use the correction factors to improve the tree-level estimates of many cross sections. For example, in dilepton production via the Drell-Yan process in nucleon-nucleon collisions, the correction factor $K^{(i)}$ is about 2.2 from Fig. 1 at high energies which agrees with experimental values of 1.6 to 2.8 [9]. In the quark-gluon plasma, dilepton production from the collision of quarks and antiquarks in the plasma occurs through the reactions $u\bar{u} \rightarrow l^+l^-$ and $d\bar{d} \rightarrow l^+l^-$. Because of our interpolation using Schwinger's procedure, the semi-empirical correction factors of Eq. (2.20) is applicable to these processes. The tree-level cross sections for $u\bar{u} \rightarrow l^+l^-$ and $d\bar{d} \rightarrow l^+l^-$ should be corrected by multiplying them with the correction factor $K^{(i)}$, to take into account the initial-state u - \bar{u} or d - \bar{d} color interactions and radiations. The results of Fig. 1 indicates that this multiplicative correction factor for dilepton production in the quark-gluon plasma is significantly large. It increases as the dilepton energy decreases. The action of the correction factor is to enhance the tree-level dilepton cross section by a factor of about 4 at $\sqrt{s} = 1$ GeV, and by a factor of about 3 at $\sqrt{s} = 2 - 3$ GeV.

In the process of $q\bar{q} \rightarrow s\bar{s}$ in the quark-gluon plasma, where q is a light u or d quark, the tree-level cross section should be multiplied by the correction factors $K^{(i)}(q\bar{q})K^{(f)}(s\bar{s})$ to take into account the initial- and final-state interactions and gluon radiations. Because these reactions take place when the quark and antiquark are in the color-octet state, the interaction is repulsive, and the two correction factors are less than unity. The deviation from unity is great near the $s\bar{s}$ threshold. Another application of the present investigation is the use of the correction factor to provide a better estimate of the production of top quarks by the e^+e^- annihilation, which will be the subject of a separate publication [35].

In the plasma, the color charge of the constituents of the plasma is subject to Debye screening. We shall investigate quantitatively how the correction factors may be modified by the Debye screening. One can get a qualitative understanding whether the Debye screening may lead to a large modification on the correction factor of Eq. (2.20) by comparing the magnitudes of length scale, over which annihilation or production takes place, with the Debye screening length. If the length scale for annihilation or production is much greater

than the Debye screening length, then the color charge of one interacting particle is effectively shielded from the other particle, when the quark and the antiquark come to the region of annihilation or production. In this case, the correction factor of Eq. (2.20) calculated with no Debye screening will be much modified when the Debye screening is taken into account. On the other hand, when the length scale for annihilation is much smaller than the Debye screening length, as in the case of dilepton production in the quark-gluon plasma, the effect of Debye screening is small. Accordingly, the Debye screening will not significantly modify the correction factor of Eq. (2.20) for dilepton production, and for the reaction $q\bar{q} \rightarrow Q\bar{Q}$ with $\sqrt{s} \gg 0.25$ GeV. The Debye screening is important for the reaction $q\bar{q} \rightarrow Q\bar{Q}$ with $\sqrt{s} \ll 0.25$ GeV. Therefore, it remains reasonable to apply the present results of the correction factor without considering Debye screening to a substantial domain of reactions in the quark-gluon plasma.

An important part of the dynamics in the plasma or partons involves the reaction with gluons. The evaluation of the complete set of reaction cross sections in the plasma or partons will require the investigation of next-to-lowest order corrections to gluon reaction processes, which will be the subject of our future studies. Just as for a $q\bar{q}$ pair, initial- and final-state gluon-gluon and gluon-quark interactions are expected to lead to large corrections to cross sections for reactions involving gg or gq pairs. A gg pair can form many different color multiplets and the interaction between a gluon and another gluon is attractive when they are in their color-singlet and color-octet states. It has been argued that gluon dynamics may be described as massive spin-1 fields with the mass generated dynamically through strong gluon-binding forces [36]. Investigations on the gluon-gluon and gluon-quark correction factors using a gluon exchange potentials as in [36,37] will be of great interest.

Future refinement of the present investigation can be carried out by including the variation of the coupling constant in modifying the spatial dependence of the color potential between a quark and its antiquark partner in the plasma. The correction factor we have obtained is based on an approximate simplified description by treating the two-body problem as a single-particle problem with the proper relativistic kinematics. A better two-body

Dirac equation based on Crater and van Alstine's constraint dynamics [26] has already been worked out and used to examine electromagnetic interactions [27]. Its application to the present problem involving a quark and an antiquark in a Coloumb-like interaction will be of great interest. Furthermore, the solution of Eq. (6) is actually obtained by neglecting the $O(\alpha_{\text{eff}}^2/r^2)$ term in the second-order Schrödinger equation derived from the single-particle Dirac equation [24]. It will be useful to include this term in future investigations.

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FIGURES

FIG. 1. The vertex correction factor $K^{(i)}$ for the annihilation of a $q\bar{q}$ pair of various flavors. The upper three curves are for color-singlet states and the lower three curves for color-octet states.

FIG. 2. The vertex correction factor $K^{(f)}$ for the production of a $q\bar{q}$ pair of various flavors. The upper three curves are for color-singlet state and the lower three curves for color-octet states.

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Correction factor for $q\bar{q}$ production

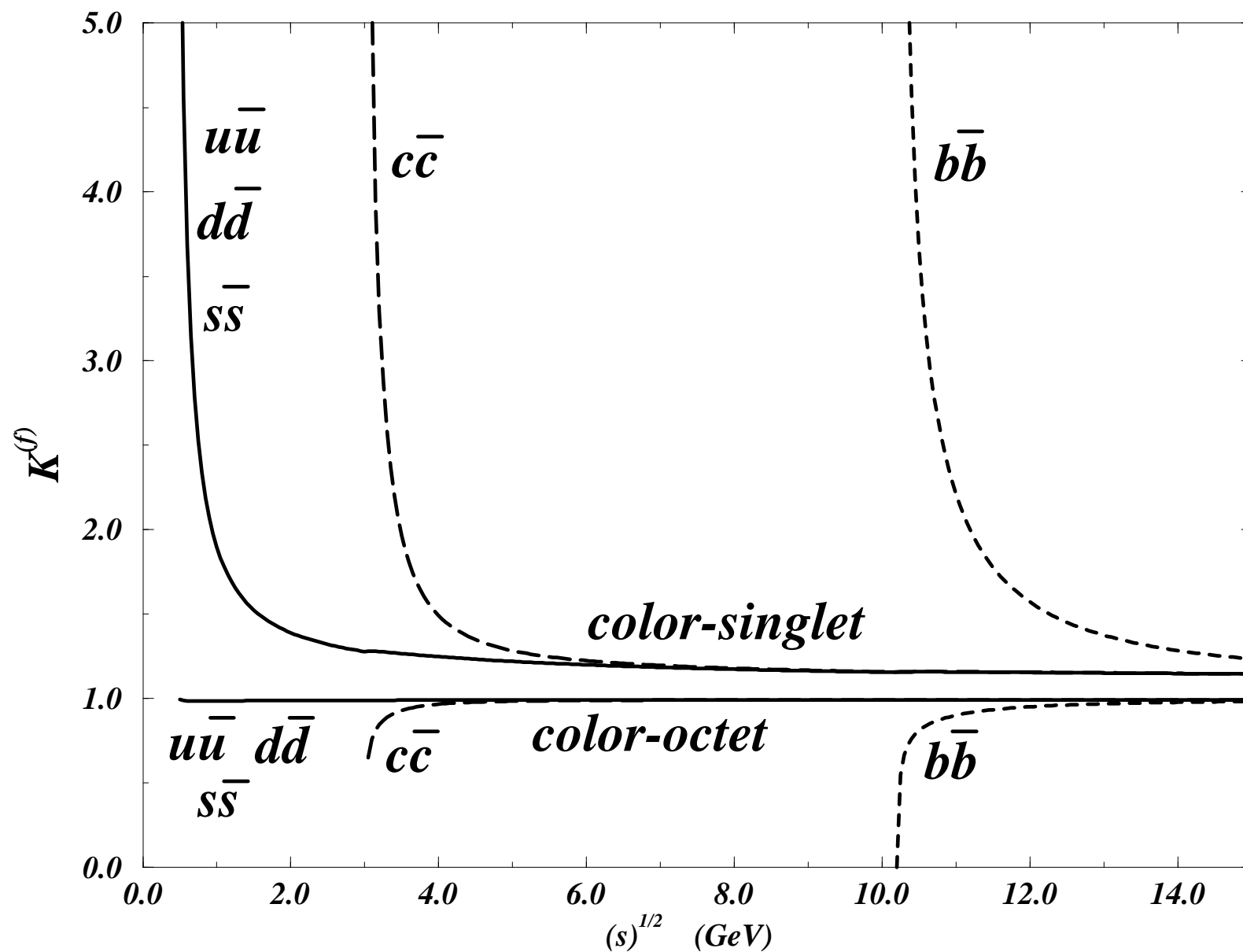


Fig. 2

Correction factor for $q\bar{q}$ annihilation

